Eur. Phys. J. C 12, 627–631 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000291

THE EUROPEAN
PHYSICAL JOURNAL C

A kinematically complete measurement of $K^+ \to \pi^+\pi^0\pi^0$ decays

The KEK-PS E246 Collaboration

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Received: 16 October 1999 / Published online: 27 January 2000 – © Springer-Verlag 2000

Abstract. In a kinematically complete setting, the $K^+ \to \pi^+\pi^0\pi^0$ decays of stopped K^+ were measured. In the frame work of the Weinberg polynomial expansion, the coefficients up to quadratic terms were deduced. The results are in good agreement with previous measurements and consistent with CP conservation expectations. The empirical $\Delta I = 1/2$ rule is violated.

1 Introduction

The non-leptonic decays of K-mesons have attracted considerable attention in the literature [1]. In particular, the $K \to 3\pi(\mathrm{K}_{\pi 3})$ decays provide information on various topics such as CP violation, $\Delta I = 1/2$ rule, and chiral symmetry breaking, etc. There are six distinct modes of $K_{\pi 3}$ decays: $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$, $K^\pm \to \pi^\pm\pi^0\pi^0$ and $K^0_{L,S} \to \pi^+\pi^-\pi^0$, herein labeled as $\mathrm{K}^{(\pm)}_{\pm\pm\mp}$, $\mathrm{K}^{(\pm)}_{00\pm}$ and $\mathrm{K}^{0(L,S)}_{+-0}$, respectively.

Weinberg [2] proposed a polynomial expansion of the decay matrix elements, written in the form

$$|M|^2 \propto 1 + gY + hY^2 + jX + kX^2 \tag{1}$$

where the kinematic variables are defined as

$$X = \frac{s_1 - s_2}{m_\pi^2} \quad \text{and} \quad Y = \frac{s_3 - s_0}{m_\pi^2}. \tag{2}$$

with $s_i = (p_\mathrm{K} - p_i)^2$, i= 1, 2, 3 corresponding to the three pions with s_3 referring to the odd pion. Also, $s_0 = (s_1 + s_2 + s_3)/3$. A few physics principles become transparent from this expansion.

For $\mathrm{K}^{(\pm)}_{00\pm}$ decays, Bose symmetry requires that j=0, since s_1 and s_2 refer to identical pions. While the CP conservation requires that $g_{++-} = g_{--+}$ and $g_{00+} = g_{00-}$, the empirical selection rule $\Delta I = 1/2$ stipulates that $g_{\pm,\pm,\mp} = \frac{1}{2} g_{00\pm}$.

In this paper, for the first time, the results of a kinematically complete measurement for the K^+_{00+} are presented. The aim was to determine the coefficients of the polynomial expansion.

2 Experimental

The measurement was performed using the apparatus constructed to search for T-violation in $K^+ \to \pi^0\mu^+\nu$ decay [3,4] at the 12 GeV proton synchrotron facility of the High Energy Accelerator Research Organization (KEK), Tsukuba, Japan. The experimental arrangement is shown in Fig. 1, and is described in detail elsewhere (see for example, [4,5]). Briefly, 660 MeV/c K^+ are slowed in a BeO degrader and stopped in an active target of 256 scintillating fibers each of 5×5 mm^2 cross section. Charged particles are momentum analyzed in the 12 sector superconducting toroidal spectrometer of solid angle of ~ 1 sr. The energies and angles of the photons from π^0 decays are measured by the CsI calorimeter, the details of which are shown in the inset of Fig. 1. It is an assembly of 768 CsI(Tl) crystals, each about 13 radiation lengths in thickness [5]. It covers a solid angle of 3π steradians, with openings for the beam entry and exit and gaps for the charged particles entry to the spectrometer.

The K$_{\pi 3}$ decay study required a trigger condition different from that of the main experiment. We employed as trigger:

$$C_{\mathrm{K}} \otimes \sum_{i=1}^{12} (\mathrm{Fid}_i \otimes \mathrm{TOF2}_i) \otimes \mathrm{n}_\gamma . \tag{3}$$

The Cerenkov condition C_{K} ensures that the beam particle is a K^+ meson, rather than a π^+ meson. The latter outnumber the former by a factor of 7. The coincidence between C_{K}, delayed by 3 ns, and the fiducial counters (Fid$_i$) for the charged particles eliminates the triggers due to decays in flight. The time of flight signal from the corresponding gap (TOF2$_i$) completes the charged particle trigger condition. In addition, the trigger requires $n \geq 2$ photons with $n_\gamma = 2$ for $K_{\pi 2}$ measured for calibration purposes and $n_\gamma > 2$ enhances the K$_{\pi 3}$ events in the data sample and suppresses triggers from other decays such as $K_{\mu 3}$.

The K$_{\pi 3}$ data were collected for three spectrometer field settings $B = 0.45$, 0.65 and 0.9 Tesla. It was intended to serve as a consistency check with regard to spectrometer acceptance and the dependences on the beam stopping position and energy loss in target, etc.

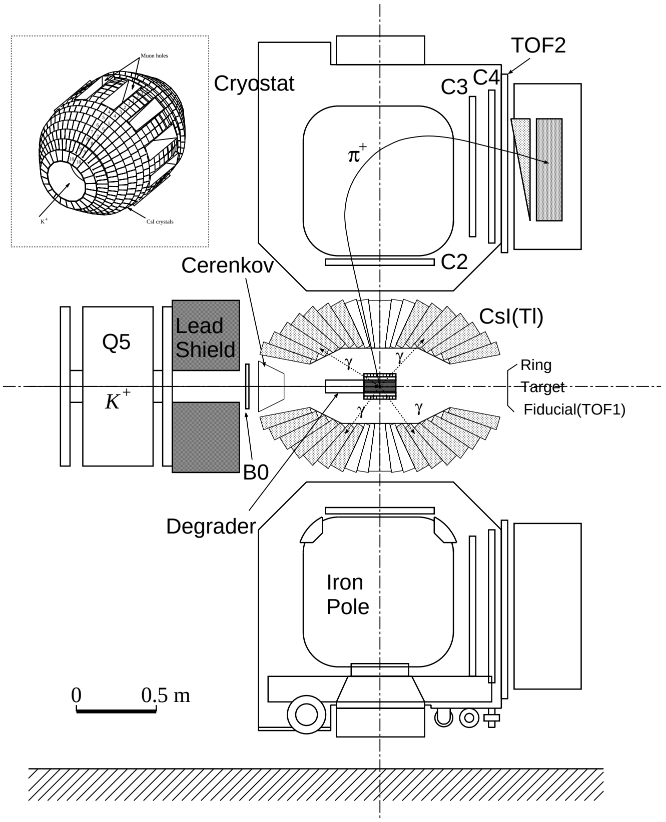

Fig. 1. Cross sectional view of the E246 setup. The layout of the CsI(Tl) assembly is shown in the inset

3 Data analysis

The $K_{\pi 2}$ ($K^+ \to \pi^+\pi^0$) channel, a two body final state with mono-energetic decay products, was used for the calibration purposes and also to verify the reliability of the Monte Carlo program. For photon energy reconstructions, the crystals with energy deposits of more than 10 MeV were included. The photon energy was determined by adding the energy deposits in a cluster of crystals (3×3 in size) from the center crystal. The Monte Carlo code is a part of the elaborate simulation program developed for the main T-violation experiment. It incorporated the geometry of the experimental arrangement and the physical interactions in the beam, the target and the detector system. The measured invariant mass and energy distributions of the π^0s are found to be in good agreement with the Monte Carlo simulation results.

The K$_{\pi 3}$ data analysis proceeds in two stages. The first stage is a selection of good events. For this purpose, we require a π^+ meson in coincidence with four photons (from the decays of two π^0s) with the followings conditions, which are specific to the decay at rest: a) the total emitted energy is equal to the K mass, m_{K},

$$E_{\mathrm{TOT}} = E_{\pi^+} + \sum_{i=1}^{4} E_{\gamma_i} = m_{\mathrm{K}}, \tag{4}$$

and b) for the decay at rest the total momentum $\mathbf{P}_{\mathrm{TOT}}$ is zero, i.e.

$$\mathbf{P}_{\mathrm{TOT}} = \mathbf{p}_{\pi^+} + \sum_{i=1}^{4} \mathbf{p}_{\gamma_i} = 0, \tag{5}$$

which can equivalently be written as

$$\begin{aligned} \Delta P = |\mathbf{p}_{\pi^+}| - |\textstyle\sum_{i=1}^{4} \mathbf{p}_{\gamma_i}| = 0 \\ \theta_{\pi^+,4\gamma} = 180^\circ , \end{aligned} \tag{6}$$

where $\theta_{\pi^+,4\gamma}$ is the opening angle between $\mathbf{p}_{\pi^+}$ and the net momentum vector of four photons, i.e. $\sum_{i=1}^{4} \mathbf{p}_{\gamma_i}$.

After the π^+ mesons were selected by the particle identification scheme using particle tracking and time of flight

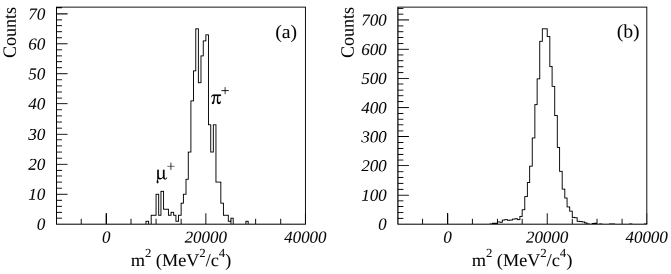

Fig. 2a,b. Mass squared plots for the charged particles, which were calculated from time of flight and momentum at the spectrometer, for the measured **a** and the simulation **b**

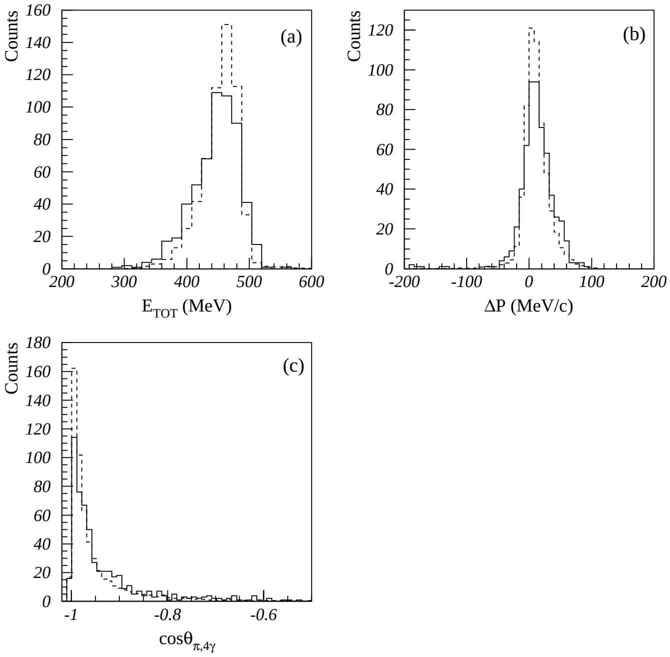

Fig. 3a–c. Kinematics of the $K_{\pi3}$ decays at rest. **a** Total energy E_{TOT} of π^+ and 4γs, **b** Momentum difference $\Delta P = p_{\pi^+} - p_{4\gamma}$ between π^+ and 4γs, **c** opening angle of $\mathbf{p}_{\pi^+}$ with $\mathbf{p}_{4\gamma}$. Solid lines are the experimental data and dashed lines are the Monte Carlo simulations

Table 1. The g and k parameters for different field settings

B (T)	g	k
0.9	0.521 ± 0.088	0.064 ± 0.056
0.65	0.477 ± 0.055	0.034 ± 0.023
0.45	0.584 ± 0.070	0.073 ± 0.054
weighted mean	0.518 ± 0.039	0.043 ± 0.020

in the spectrometer gap, the $K_{\pi3}$ events were selected from the above two criteria.

Figure 2 shows the mass squared plots for the charged particles, which were calculated from time of flight and momentum in the spectrometer, for both the measured (a) and the simulated (b) data, respectively. Fig. 2a clearly shows the separation of π^+ and μ^+. The Monte Carlo for $K_{\pi3}$ events were generated for 3-body phase space.

Figure 3 shows E_{TOT} [Fig. 3a] and ΔP [Fig. 3b] plots for the selected events. It is observed that E_{TOT} peaks slightly lower than the kaon mass. Also, the momentum balance condition with the peak centered at zero momentum is satisfied, although the distribution is slightly asymmetric. The widths of the distributions are understood as being due to the finite resolution of the detector elements. The shift in E_{TOT} and the asymmetry in ΔP are due to the electromagnetic shower leakage in our CsI crystal arrangement. For our detector, this leakage typically underestimates photon energy by 5% with some low energy tail. Our Monte Carlo routines include this leakage effect and simulations are in good agreement with the experimental data. Fig. 3c shows the opening angle distribution of the π^+ with the resultant of the four photon direction. A sharp peak at $\cos\theta_{\pi^+,4\gamma} = -1$ is observed as required for $K_{\pi3}$ decays. The final event selection satisfied $-50 < \Delta P < 70(MeV/c)$ and $\cos\theta_{\pi^+,4\gamma} < -0.8$. These cuts leave 143, 503 and 169 events for 0.45T, 0.6T and 0.9T, respecitvely.

If one is satisfied with the determination of the g and h parameters of the polynomial expansion, (1), which depend on the momenta of π^+ mesons only, this analysis would have been sufficient. However, in order to determine the k parameter, it is necessary to determine which pairs of photons, out of the three possible combinations in each event, constitute the π^0s. To this end, we define a parameter Q as

$$Q = (m_{\gamma_i\gamma_j} - m_{\pi^0})^2 + (m_{\gamma_k\gamma_l} - m_{\pi^0})^2, \qquad (7)$$

where $m_{\gamma_i\gamma_j}$ and $m_{\gamma_k\gamma_l}$ are the invariant masses of γ_i, γ_j, and γ_k, γ_l pairs ($i \neq j \neq k \neq l = 1,4$), respectively and $m_{\pi^0} = 135\,\mathrm{MeV/c^2}$ is the rest mass of pion. The combination with the minimum value for Q is considered as the correct pairs. Figures 4a and b show the invariant masses of the two π^0s thus identified. The π^0 with higher energy is labeled #1 and the other #2. They show smooth distributions with peaks at m_{π^0}. Also shown are the Monte Carlo results as dotted lines, subject to the same criterion as the experimental data. Good agreement between the experiment and Monte Carlo is evident. The same aspect is further examined from 2-D plots of mass spectra of the two π^0s. Figure 4c shows a plot of $m_{\pi^0}(1)$ versus $m_{\pi^0}(2)$. There is a clear clustering of events at $m_{\pi^0}(1) \sim m_{\pi^0}(2) \sim 135\,\mathrm{MeV/c^2}$. For comparison, the plot of invariant masses of the rejected combinations are shown in Fig. 4d. The Figs. 4e and 4f show the corresponding plots from the Monte Carlo simulation. Again, one sees that Monte Carlo and experiment are in very good agreement. This gives confidence in the pairing criterion for selecting the two π^0s on an event by event basis. From the distribution of unselected pair candidates, it is estimated that the residual bias due to wrong pairing is less than 2%.

4 Results

To deduce the physics parameters of interest, the experimental data (EXP) is compared with the Monte Carlo

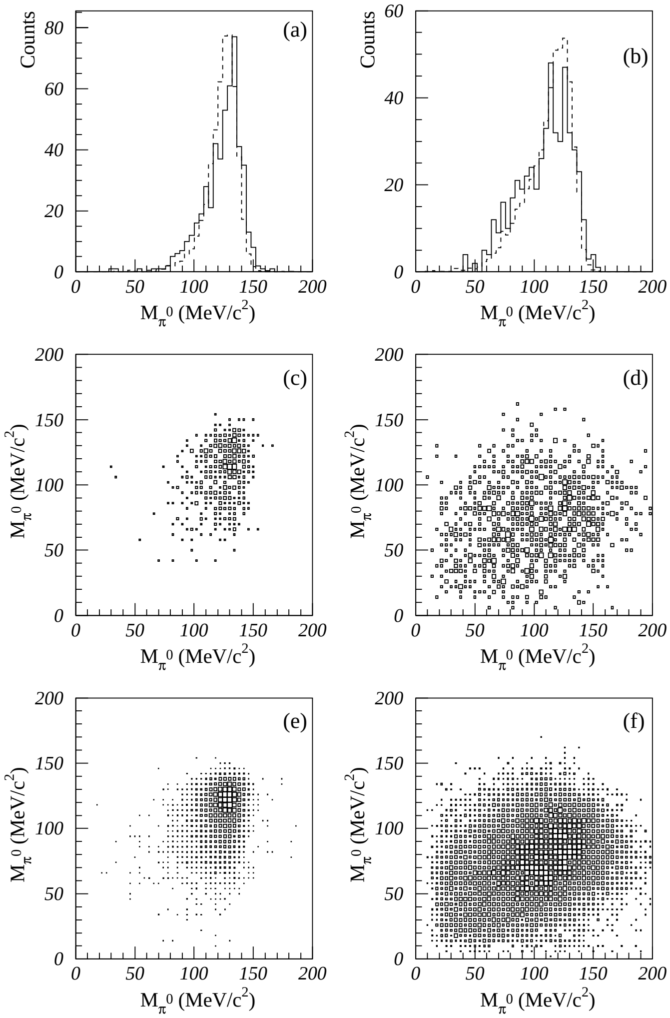

Fig. 4a–f. Invariant mass plots for the selected π^0 pair in K$_{\pi3}$ events. **a** is invariant mass plot of the γ pair which has larger energy than that of the ones in **b**. **c** shows the scatter plot of the invariant masses of best π^0 pair. **d** shows the same plot for the unselected π^0 pair candidates. **e** and **f** are the plots of Monte Carlo simulations with the criteria of **c** and **d**, respectively

(MC) events for the K$_{\pi3}$ decays. Figure 5 shows the plots of yield ratio $dR(s_3-s_0) = dN^{\mathrm{EXP}}/d(s_3-s_0)/dN^{\mathrm{MC}}(s_3-s_0)$ along with a straight line fit. In each plot, the data at extreme ends have larger errors, because they are at the edge of available phase space. Table 1 shows the deduced 'g' parameter for each field setting and the results agree within one standard deviation. It is quite heartening to note that there is no systematic change with spectrometer field, which indicates that the errors due to the beam stopping distribution, energy losses of π^+ particles in the target etc., are quite small. The narrow region of (s_3-s_0) available in this experiment did not permit investigation of the weak quadratic term (h). The weighted mean of the three field setting measurements for the coefficient $g = 0.518 \pm 0.039$.

As mentioned before, it is known that the coefficient 'j' of the linear term in (s_2-s_1) should be zero due to Bose symmetry condition. We, therefore, fit the $dR(s_2-s_1) = dN^{\mathrm{EXP}}/d(s_2-s_1)/dN^{\mathrm{MC}}/d(s_2-s_1)$ result for a quadratic term(k). Figure 6 shows the plots of the yield ratio $dR(s_2-s1)$. It is worth noting that the range of (s_1-s_2) covered by the current experiment is very close to the threshold. At each field setting, the data is well reproduced by a quadratic dependence on (s_1-s_2). The fit results are also tabulated in Table 1. A non-zero value

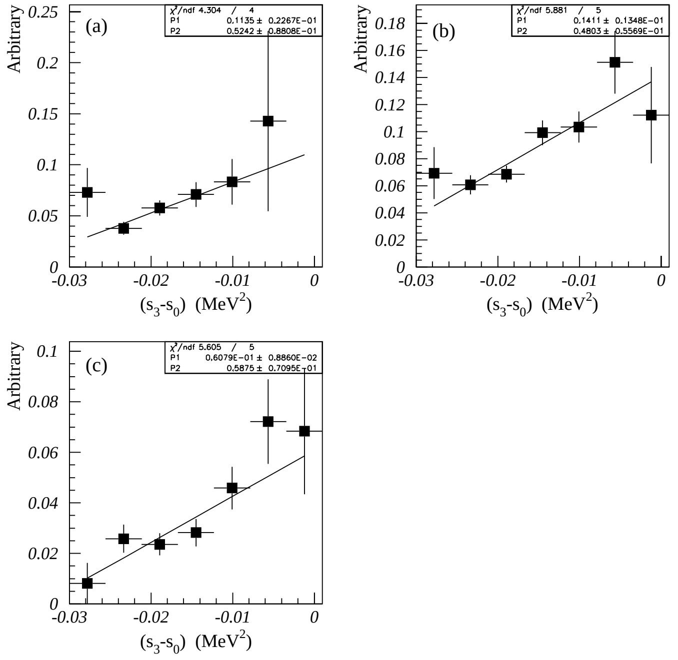

Fig. 5a–c. $dR(s_3-s_0) = dN^{\mathrm{EXP}}/d(s_3-s_0)/dN^{\mathrm{MC}}/d(s_3-s_0)$ distributions for spectrometer field settings **a** 0.9 T; **b** 0.65 T; and **c** 0.45 T. The straight lines are the best fits to the data

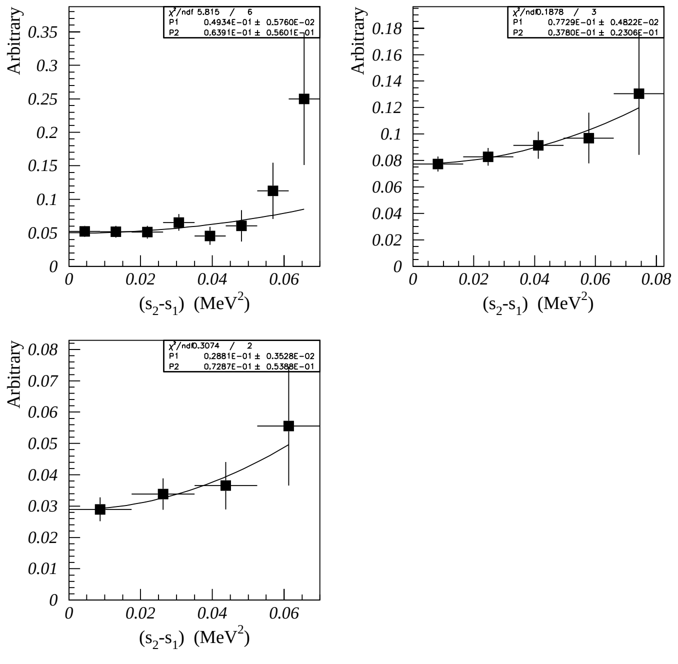

Fig. 6a–c. $dR(s_2-s_1) = dN^{\mathrm{EXP}}/d(s_2-s_1)/dN^{\mathrm{MC}}/d(s_2-s_1)$ distributions for each field settings fitted with linear functional form in $(s_1-s_2)^2$. **a** for 0.9 T; **b** for 0.65 T; **c** for 0.45 T

Table 2. Summary of the measurements of *g*,*h*, *k* parameters of Weinberg's expansion in the $K^+_{\pi 3}$ Decays

Ref.	g	h	k
Present	0.518 ± 0.039	–	0.043 ± 0.020
Batusov et al. [6]	0.704 ± 0.012	0.104 ± 0.014	0.0197 ± 0.0045
Braun et al. [7]	0.670 ± 0.054	0.152 ± 0.082	-0.02 ± 0.04
Bertrand et al. [8]	0.806 ± 0.220	0.164 ± 0.121	—
Sheaff [9]	0.630 ± 0.038	0.041 ± 0.030	—
Smith et al. [10]	0.510 ± 0.060	0.009 ± 0.040	—
Aubert et al. [11]	0.67 ± 0.06	-0.01 ± 0.08	—
Davison et al. [12]	0.544 ± 0.048	0.026 ± 0.050	—
World average [a]	0.673 ± 0.010		
World average[b]	0.587 ± 0.019		

[a] This estimate is the weighted mean of all values quoted above.

[b] The results of [6] are not included in this world average estimate. See text for details.

$k = 0.043 \pm 0.020$, is obtained from the weighted mean of the three measurements.

Table 2 presents the results of $K^+ \to \pi^+\pi^0\pi^0$ measurements available to date, including the present results. It is to be remarked that 'g' values of all experiments, with the exception of Batusov et al.,[6] are in agreement. As Batusov et al. measured the in flight decays of 10 GeV/c kaons, it is tempting to attribute the discrepancy to a possible energy dependence of 'g' parameter. This may not, however, be entirely correct because the result of Smith et al. [10], which employed in flight decay of 5 GeV/c kaons, is in excellent agreement with the present result. The world average from all the data, except that of Batusov et al. [6], gives $g = 0.587 \pm 0.019$. This result is in very good agreement with the $g = 0.582 \pm 0.021$ for the CP symmetry partner $K^- \to \pi^-\pi^0\pi^0$ [13]. If we include Batusov et al's result in the world average estimate, we get $g = 0.673 \pm 0.010$, about three standard deviations away from the CP symmetry expectation. The 'g ' parameters for the decays are $g_{++-} = -0.2154 \pm 0.0035$, and $g_{--+} = -0.217 \pm 0.007$, which predict, from $\Delta I = 1/2$ rule, $g_{+00} = 0.43$ for the decay mode of our interest. Clearly, this rule is not obeyed.

Prior to our work, there were only two other measurements which attempted to determine the parameter 'k' in this channel. The result of Braun et al. [7] was obtained from a simultaneous fit of three parameters, without π^0 reconstruction. The only other work is due to Batusov et al [6]. Within the errors, all these results are compatible. Both our result and that of Batusov et al show a non-zero 'k' parameter, while that of Braun et al is consistent with k=0.

5 Summary and conclusions

We have measured, in a kinematically over determined setting, the $K^+ \to \pi^+\pi^0\pi^0$ decay parameters. The linear term in Weinberg expansion is determined and the world data now fixes it to $g = 0.587 \pm 0.019$, comparable in precision and in good agreement with the CP symmetry partner decay of $K^- \to \pi^-\pi^0\pi^0$ with $g = 0.582 \pm 0.021$. It is found that the quadratic term in $(s_2 - s_1)$ is $k = 0.043 \pm 0.020$, a small but non-zero term.

Acknowledgements. We would like to acknowledge financial support from several granting agencies (Ministry of Education, Science, Sports, and Culture-Japan, JSPS-Japan, Ministry of Science and Technology of Russia, Russian Foundation for Basic Research, NSERC-Canada, KOSEF-Korea, NSF-USA and DOE-USA). We would also like to thank Professors K. Nakai and K. Nakamura as well as the technical and support staff of KEK-PS.

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